In-Situ Inc.

Water Level Accuracy and Correcting for Errors due to Gravitational Acceleration and Liquid Density

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In-Situ pressure transducers are manufactured in the United States and made to the English specification of integer pounds per square inch (PSI). When converting from PSI units to meters or feet of water (H_2O), several conversion factors are needed. One of these is the **acceleration due to gravity**. The acceleration due to gravity that an object experiences in a specific location is a function of both latitude and altitude. The average gravitational acceleration (*g*) value is 9.80665 m/s^2, but this value actually varies depending on location. Incorrect assumptions can introduce water level errors as large as 0.27% at the equator and at sea level. Another conversion factor needed in the calculation is the **density** of the aqueous solution. A significant error is introduced when density (ρ) is approximated as 1,000 kg/m^3 (1.000 g/cm^3), the density of pure water at 4°C. Pure ground water at 20°C actually has a density of 998 kg/m^3, and this difference corresponds to an error of 0.20%.

CONVERSION TO METERS OF WATER

Conversion of pressure expressed in PSI units to water level or depth expressed in meters of H_2O requires that the pressure be converted to the SI unit Pascal (Pa).

$$1 \text{ PSI} = 6.894757 \text{ kPa} = 6894.757 \text{ Pa.}$$

By definition, a Pa = N/m^2 and a Newton (N) = kg m/s^2, so a Pa = kg m/s^2/m^2 (kg/s^{2}m), giving:

$$1 \text{ PSI} = 6894.757 \text{ kg/s}^2\text{m.}$$

Obtaining the pressure in **m of H_2O** from the unit **kg/s^{2}m** requires using the liquid's density (r) in **kg/m^3** and the gravitational acceleration (*g*) in **m/s^2**.

The derived formula is:

$$\text{m of } H_2O = \frac{\text{PSI} \times 6894.757 \text{ Pa/PSI}}{g \times \rho} \quad (1)$$

Substituting the correct values for gravity and density will give an accurate value for water level or depth expressed in m of H_2O for a pressure reading obtained in PSI.

OBTAINING A CORRECT VALUE FOR *g*

Gravitational acceleration varies from 9.78036 m/s^2 at the equator to 9.83208 m/s^2 at the poles. It also decreases in value by 0.003086 m/s^2 for every km above sea level.

The following formula can be used to determine *g* in m/s^2 for a specific location:

$$g = 9.780356 (1 + 0.0052885 \sin^2 \phi - 0.0000059 \sin^2 2\phi) - 0.003086 H \quad (2)$$

where ϕ is the latitude in degrees and *H* is the altitude above sea level in km (Jursa, A.S., Ed., *Handbook of Geophysics and the Space Environment*, 4th ed., Air Force Geophysics Laboratory, 1985, pp. 14-17).

OBTAINING A CORRECT VALUE FOR ρ

Water density is a function of temperature and the quantity of dissolved minerals or contaminants. If the water has few minerals or contaminants, the density is largely dependent upon temperature. The following table gives the density of pure water as a function of temperature in g/cm^3. The values are computed from the relative values by Thiesen, Scheel and Disselhorst (1900), and the absolute value at 3.98°C by the International Bureau of Weights and Measures (1910).

Temp. (°C)	Density (g/cm^3)	Temp. (°C)	Density (g/cm^3)	Temp. (°C)	Density (g/cm^3)
1	0.999900	11	0.999605	21	0.997992
2	0.999941	12	0.999498	22	0.997770
3	0.999965	13	0.999377	23	0.997538
4	0.999973	14	0.999244	24	0.997296
5	0.999965	15	0.999099	25	0.997044
6	0.999941	16	0.998943	26	0.996783
7	0.999902	17	0.998774	27	0.996512
8	0.999849	18	0.998595	28	0.996232
9	0.999781	19	0.998405	29	0.995944
10	0.999700	20	0.998203	30	0.995646

If the water has many dissolved minerals or contaminants, however, it is necessary to physically measure the density to obtain an accurate value for ρ. A hydrometer can be used to determine the specific gravity of the water and this in turn can be converted into density in kg/m^3. The hydrometer must be accurate to ± 0.0005 in order to be useful. Simply obtain a sample of the water to be monitored and read the specific gravity on the hydrometer. If the specific gravity is 0.9985, then the density is 0.9985 g/cm^3 (998.5 kg/m^3).

Density can also be determined with an accurate tape measure and a pressure transducer. The measuring tape is securely attached to the transducer cable. A measurement is recorded from the tape with a corresponding reading taken from the pressure transducer. The transducer is lowered deeper into the water and a second set of measurements is recorded. The density of the water in kg/m^3 is given as:

$$\rho = \frac{\text{Change in pressure on transducer}}{\text{Change in length on tape}} \times 1000 \qquad (3)$$

If the measuring tape is calibrated in meters, the pressure units from the transducer must be converted to meters of H_2O at 4°C. Versions 2.31 and earlier of Win-Situ / Data Manager use the value at 4°C to convert PSI to meters of H_2O. 1 PSI = 0.7030695 m of H_2O at 4°C assuming a density of 1,000 kg/m^3 (1.000 g/cm^3). Density measurements also assume that water temperature and therefore density in the well is homogeneous throughout its entire depth.

EXAMPLE

Consider the gravitational acceleration in Laramie, Wyoming. The latitude (ϕ) for Laramie is approximately 41° N while the altitude (H) is approximately 2.195 km (7,200 ft). Substituting into equation (2) gives g = 9.79579 m/s^2 for Laramie. Compare this to the average value of g = 9.80665 m/s^2. This represents an error of 0.11% for g relative to the average value of g. This translates into an error of 0.11% FS or 12 mm of H_2O for 15 PSI and 24 mm for 30 PSI.

The magnitude of error is less for locations around 45° latitude at sea level, since the true value of g at 45° is very close to 9.80665 m/s^2. Error increases for locations closer to the equator or the poles, as shown in the table on this page.

City	Latitude	Error introduced using average g (15 PSI)
Edmonton, Canada	54°N	7.6 mm
Amsterdam	52°N	6.4 mm
London	51°N	5.7 mm
Paris	48°N	3.3 mm
New York	41°N	4.6 mm
Tokyo	36°N	9.6 mm
Los Angeles	34°N	11.0 mm
Hyderabad, India	17°N	23.6 mm
Caracas, Venezuela	10°N	26.7 mm

The magnitude of error can be compounded if the values are not corrected for temperature and density. Assuming that the density of water is 1,000 kg/m^3 instead of the actual value, 998 kg/m^3 for pure water around 68°F (20°C), introduces an error of 0.20% relative to the true value. This translates into an error of 0.20% FS or 21 mm H_2O for a 15 PSI sensor and 42 mm for a 30 PSI sensor. If the water is warmer, say 77°F (25°C), and the density is assumed to be 1,000 kg/m^3 (it is actually 997 kg/m^3 for pure H_2O at 77°F), the error corresponding to this invalid assumption is 0.30% FS or 32 mm for a 15 PSI sensor and 64 mm for a 30 PSI sensor. Add this error to that from an invalid gravitational acceleration value at 15° N Latitude in southern India and the total error is ±0.53% FS or 57 mm for 15 PSI and 114 mm (4.5 inches) for 30 PSI!

CONCLUSIONS

A significant error can be introduced when converting PSI units into water level or depth by using incorrect values for gravitational acceleration (g) and density (ρ). Gravitational acceleration depends on location and is a function of latitude and altitude, while water density depends on temperature and dissolved impurities. A spec of 0.05% FS accuracy on a 15 PSI sensor corresponds to only 5.3 mm of H_2O at 68°F (20°C). The error introduced using average gravitational acceleration (g) instead of the true value for Laramie is more than double the amount of error allowed for a spec of 0.05% FS on a 15 PSI sensor!

Win-Situ 2000 and Win-Situ 4 allow compensation for the density of the liquid and for changes in gravitational acceleration.

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